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Numerical Simulation of the Growth of Localized Disturbances in a Supersonic Boundary Layer over a Plate with Longitudinal Slots

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This paper presents the results of numerical simulations of the growth of small amplitude localized disturbances in the boundary layer on a flat plate and on a plate with rectangular longitudinal slots at Mach number 2. The simulations were performed with the FlowVision software under flow conditions corresponding to those in the T-325 wind tunnel of the Khristianovich Institute of Theoretical and Applied Mechanics SB RAS. Cases with slots no deeper than the boundary layer thickness were considered. The localized disturbances in the boundary layer were generated by a time-pulsed, spatially localized heat supply from the plate surface. For a smooth plate, the growth of the disturbances was compared with the results of the linear stability theory. Data agreement was achieved for the grid resolution used. For the cases of the smooth surface and the plate with longitudinal slots, the evolution of disturbances in the boundary layer was analyzed in physical and wave space. In case of the smooth plate, the disturbance introduced into the boundary layer increases monotonically downstream. However, in the presence of longitudinal slots, the growth of the disturbance growth is different and depends on the depth of the slots. It has been shown that longitudinal slots can affect the stability of the supersonic boundary layer. A frequency-wavenumber analysis of the disturbance evolution revealed that longitudinal slots shift the range of the most unstable waves towards a region of higher frequencies compared to the smooth plate. This effect becomes more pronounced with increasing depth of the slots.

Keywords: numerical simulation, localized disturbances, supersonic boundary layer,laminar-turbulent transition control, surface microrelief, rectangular slots.

Introduction

The laminar-turbulent transition of high-speed boundary layers leads to a significant increase in viscous friction and heat fluxes, which can lead to serious limitations in the performance of thermal protection systems for high-speed vehicles. At low levels of disturbance, turbulence in the flow is caused by the occurrence and growth of various unstable waves in the boundary layer. These waves interact with each other and can cause the flow to become turbulent [2, 8].

One possible method for controlling the boundary layer laminar-turbulent transition is through the microprofiling of the surface of a streamlined model (porous coatings, slots, grooves, etc). The advantage of microprofiling is the minimal effect on the main flow, while it is possible to influence the stability of the boundary layer. This approach is successfully applied at low subsonic flow velocities to control the laminar-turbulent transition [3, 6, 7, 13, 14].

At high flow velocities, this approach is also used to control the flow in boundary layers. Recently, special attention has been paid to using spatially periodic slots on the surface of streamlined bodies to control the position of the laminar-turbulent transition. This method has proven effective in stabilizing disturbances of the second (acoustic) mode of instability. A review of these studies can be found in [11].

In [5], the influence of small grooves on the surface on the laminar-turbulent transition of a supersonic boundary layer of a swept wing was studied using numerical simulation and linear stability theory. In these studies, the grooves on the surface were located parallel to the leading

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edge of the wing. It was shown that such a modification of the surface affects the growth of cross-flow instability modes typical for three-dimensional boundary layers.

At low supersonic flow velocities in two-dimensional boundary layers, the main role in laminar-turbulent transition is played by the growth of the first mode of disturbances (Tollmien–Schlichting waves). The paper [18] considers the possibility of controlling the stability of a supersonic boundary layer on a flat plate using indentations on the surface located across the incident flow. Both calculations using linear stability theory and numerical simulation use only two-dimensional equations. While calculations using linear theory succeeded in stabilizing disturbances of the first modes, more accurate calculations with Navier–Stokes equations (taking into account recirculation flow inside slots and alternating expansion/compression waves induced at groove edges, which were ignored in calculations with linear theory) failed to stabilize the first mode. It should be noted that only transverse slots were considered in the work (with an orientation angle of 90 relative to the incident flow), while different orientations of the slots are of interest.

In experiments in the low-noise wind tunnel T-325 at ITAM SB RAS, described in [11], a stabilization of the development of disturbances in the boundary layer of a plate with longitudinal grooves (slots) at Mach number $M = 2$ was discovered. In the experiments, the effect of longitudinal slots periodically arranged in the direction transverse to the flow was considered. In this case, the depth of the slots was comparable to the thickness of the boundary layer, and the period in the transverse direction was 5–8 times smaller than the characteristic wavelength of the most unstable disturbances. It was found that in the presence of depressions, the natural disturbances of the boundary layer grow significantly slower than in case of a smooth plate.

A well-developed method for studying the evolution of disturbances in the boundary layer is needed to study in detail the influence of longitudinal slots on the laminar-turbulent transition in supersonic boundary layers. Most information about the stability of the boundary layer can be obtained by analyzing the development of disturbances with a wide range of frequencies and wavenumber spectra. In this context, the development of a controlled wave packet has been considered, which has been widely used in the study of laminar-turbulent transitions at supersonic speeds through both numerical simulations [4, 10, 12, 15] and experiments [9, 16, 17].

This paper presents the formulation and results of numerical modeling of the evolution of spatially and temporally localized small amplitude disturbances in the boundary layer of a smooth plate and a plate with longitudinal rectangular slots at Mach number $M = 2$. The first part of the paper describes the formulation of the problem. The second part verifies the results of the non-stationary perturbation evolution for the smooth plate – comparisons of the computational results with those obtained using linear stability theory are given. The third part presents numerical results. The spatio-temporal structure and integral characteristics of the disturbance growth are analyzed and a frequency-wavenumber analysis is performed. Finally, the main conclusions of this work are presented.

1. Simulation Set-Up

The calculations are similar to the experiments [11] carried out in the low-noise supersonic wind tunnel T-325 of the ITAM SB RAS: Mach number $M = 2$, unit Reynolds number of the flow $Re_1 = U_\infty/\nu_\infty = 6 \cdot 10^6\ m^{-1}$. The flow is described within the framework of a continuum model for a compressible, viscous, and heat-conducting gas (air). Calculations were carried out in a three-dimensional formulation using the FlowVision software package. The numerical implementation in this study utilizes the FlowVision's second-order accurate reconstruction method for convective fluxes at cell interfaces, based on a monotonic upwind scheme. Temporal integration employs an explicit scheme for convective-diffusive terms, while the pressure Poisson equation is solved implicitly [1].

Figure 1 schematically shows the calculations set-up. The length of the computational domain along the flow direction (x-axis) is 130 mm, the width along the z-axis is 20.4 mm. The lower boundary of the computational domain corresponds to the plate with the slots. From the leading edge, which corresponds to the input boundary of the computational domain, to $x = 53$ mm ($Re_x = Re_1 \cdot x = 318000$) there is a flat section of the plate. Downstream there are slots that are aligned along the incident flow and arranged periodically in the transverse direction across the entire width of the computational domain. The width of the slots is 0.6 mm, and the period of their arrangement along z is 1.2 mm. Three cases are studied in this paper: a smooth plate, a plate with slots of $h = 0.18$ and 0.5 mm depth ($Re_h = Re_1 \cdot h = 1080$ and 3000). The depth of the slots is comparable to the thickness of the boundary layer. When constructing the computational domain, it was taken into account that the compression wave from the leading edge and the rarefaction waves from the slots fall on the output boundaries of the computational domain and are not reflected into the boundary layer. Figure 1 shows the pressure distribution in the xy-plane.

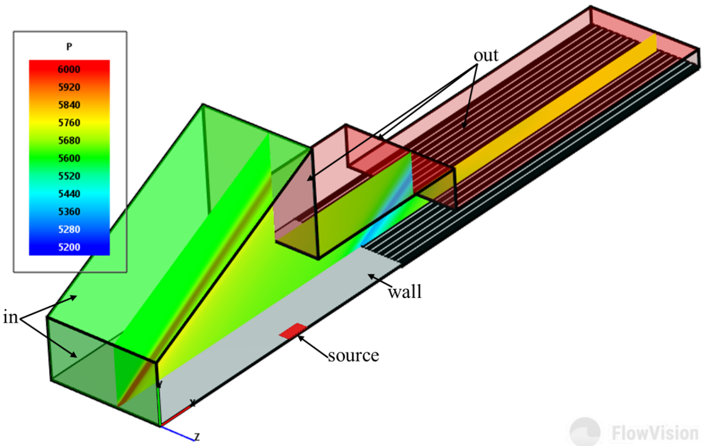


Figure 1. Computational domain

The problem was solved in a symmetric formulation relative to the xy-plane at $z = 0$. When analyzing the results, the numerical data were supplemented by mirroring along z relative to $z = 0$.

The following parameters of the incoming flow were defined at the input boundaries of the computational domain: Velocity along x 501 m/s, flow temperature 156 K, pressure 5590 Pa. These flow parameters correspond to the Mach number $M = 2$ and the unit Reynolds number $Re_1 = 6 \cdot 10^6\ m^{-1}$. The following conditions were set at the output boundaries: Temperature gradient zero, for the velocities and pressure at the boundary, values equal to the value at the center of the boundary cell are specified. This boundary condition is known as the supersonic outlet and is built into FlowVision. The symmetry condition was set on the side boundaries (in the yz-plane). The no-slip boundary condition and the zero heat flux condition were set on the slotted plate. To introduce controlled disturbances in the boundary layer, a source region was selected on the smooth section of the plate; the disturbances were generated by a non-zero heat flux in the source region for a limited time.

The structure of the computational grid on which the main studies were carried out is shown in Fig. 2. The grid was refined along y in the plate region in order to achieve a high resolution of the flow in the boundary layer. In the boundary layer region, the cell size was 0.01 mm, and there were at least 60 cells per boundary layer thickness in the region of the main flow analysis. Along the flow, the main grid step was 0.5 mm. Preliminary calculations showed that it was additionally necessary to refine the grid along the x-coordinate in the region of the start of the slots. Along the z-coordinate, the cell size was set to 0.05 mm, so that there were 24 cells per slot period. It should be noted that for the conditions of modeling in the boundary layer on a smooth plate according to linear stability theory, the wavelength of the most unstable disturbances is 6–12 mm. Studies on the convergence of the steady-state solution on the grid showed that the grid used gives an accurate mean flow. Ultimately, the number of cells was about 20 million and varied depending on the depth of the slots.

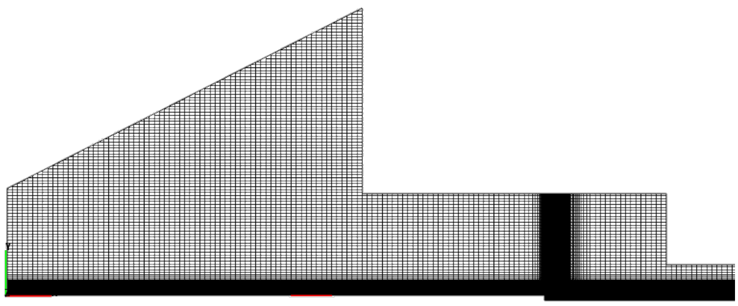


Figure 2. Computational grid

The calculation was carried out in two stages: at the first stage the steady flow was calculated, at the second stage the non-stationary calculation with the introduction of disturbances into the boundary layer. Artificial disturbances were generated into the boundary layer by changing the boundary conditions in a localized area on the plate surface (source in Fig. 1). On the plate surface, the condition for the heat flux q_w was defined at time zero for time t_s in the source region:

$$q_w(x, z, t) = q_0 \cdot \sin(\pi \frac{t}{t_s}) \cdot \cos^3(\frac{\pi}{2} \cdot \frac{x - x_s}{r_s}) \cdot \cos^3(\frac{\pi}{2} \cdot \frac{z - z_s}{r_s}), \quad (1)$$

where $x_s = 30$ mm ($Re_{x_s} = 180000$) and $z_s = 0$ are the coordinates of the center of the source, $r_s = 2$ mm is the size of the source, $t_s = 25\ \mu s$ is the duration of the impact in the boundary layer, $q_0 = 5000\ W/m^2$ is the amplitude. This condition was set in the region $(x_s \pm r_s, z_s \pm r_s)$ for the time t_s. After the time t_s, the condition for the heat flux zero was restored. The location of the source and the operating time were chosen so that they could be implemented in the experiment. The amplitude of the impact was chosen so that the evolution of the excited disturbances in the boundary layer in the investigation area was linear, but the amplitude of the disturbances was clearly above the level of the numerical pulsations.

During the calculations, the instantaneous flow parameters in the zy-sections were recorded with a step in the longitudinal direction of 5 mm in the range $x = 40$–120 mm ($Re_x = 240000$–720000). The recording was done every 500 ns, and the total time of "flight" of the disturbance from the source did not exceed 500 μs.

The calculations were carried out using the FlowVision software package on the ITAM SB RAS ("Mechanics" Shared-Use Centre) cluster. Two computing nodes were used. Each node was equipped with two 64-core AMD EPYC 7763 processors and 512 GB RAM. The computation of the second stage (non-stationary problem) for each case took approximately one week.

In order to analyze the development of local disturbances by sources, pulsations of the longitudinal component of the mass flow are considered, which are normalized to the value in the oncoming flow:

$$m'(x,y,z,t) = \frac{(\rho U(x,y,z,t) - \rho U(x,y,z,0))}{\rho U_\infty} \cdot 100\%, \tag{2}$$

where ρU_∞ is the mass flow in the oncoming flow.

In order to determine the influence of slots on the growth of a localized disturbance in the boundary layer in physical space, the evolution of the dispersion of mass flow pulsations was calculated using the data obtained, calculated in the yz planes and normalized to the value in the initial section ($x_0 = 40$ mm):

$$S_m(x) = \sqrt{\frac{\sum_{y,z,t} m'(x,y,z,t)^2}{\sum_{y,z,t} m'(x_0,y,z,t)^2}}. \tag{3}$$

The article deals with the downstream development of small amplitude disturbances at Reynolds numbers corresponding to the early stage of the laminar-turbulent transition. The early stages of the transition are described by linear stability theory. Therefore, the use of the wave approach in the data analysis is necessary to determine the influence of slots on the stability of the boundary layer. To determine the frequency-wavenumber characteristics of the disturbance evolution, the Fourier transform was carried out in time and space:

$$m_{f\beta} = \frac{\sqrt{2}}{T \cdot Z_0} \sum_{z,t} m'(z,t) \cdot \exp(i2\pi f t - i\beta z)\Delta t \Delta z, \tag{4}$$

their $T = 500$ μs, $Z_0 = 1$ mm are normalization coefficients.

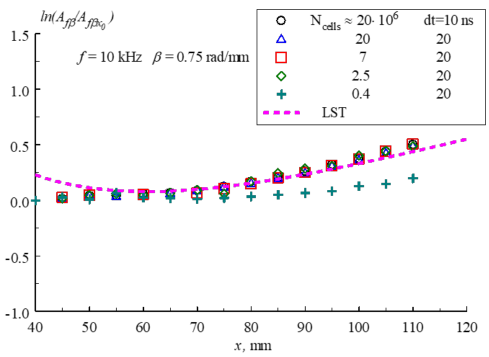
The amplitudes and phases were determined for waves with different frequencies and wavenumbers. The amplitude was defined as the modulus of the Fourier transform and the phase as an argument. To determine the dispersion relation, the longitudinal wavenumber α_r was also estimated:

$$A_{f\beta} = |m_{f\beta}|,\ \phi = \arg(m_{f\beta}),\ \alpha_r = \frac{d\phi}{dx}. \tag{5}$$

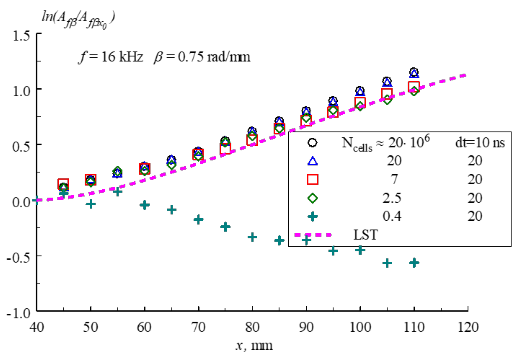
2. Verification of Calculations

The numerical simulation of disturbance evolution in the boundary layer requires a careful approach. The studies performed with different computational settings have shown that both the computational grid and the time step can have a significant impact on the growth of the disturbances. Moreover, for a correct modeling of the perturbation evolution in the early stage of the transition, it is necessary that the amplitude of the introduced perturbations is small and their evolution is consistent with the linear stability theory.

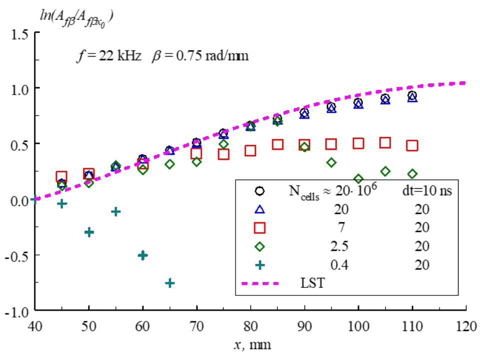
In this work, the disturbance growth is compared with the results of the linear stability theory in order to verify and optimize the calculation method. For a laminar two-dimensional supersonic Blasius boundary layer, the calculations of the disturbance growth are performed within the framework of linear stability theory (LST). From the numerical simulation data for the case of the smooth plate, the disturbance amplitudes are determined for different values of the longitudinal coordinate. Waves with a wavenumber corresponding to the most growing waves are considered. Figure 3 shows the results for the frequencies f = 10, 16 and 22 kHz and the wavenumber of $\beta = 0.75$ rad/mm. The calculations were carried out with different grid resolutions, whereby the grid structure remained approximately the same (Fig. 2).



(a) 10 kHz



(b) 16 kHz



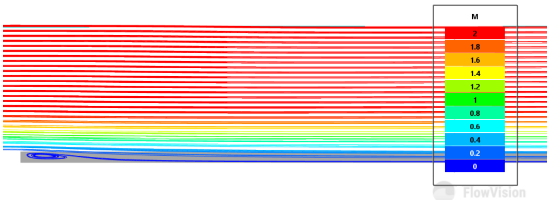
(c) 22 kHz

Figure 3. Comparison of the results of the numerical simulation of disturbance growth with the results of linear stability theory

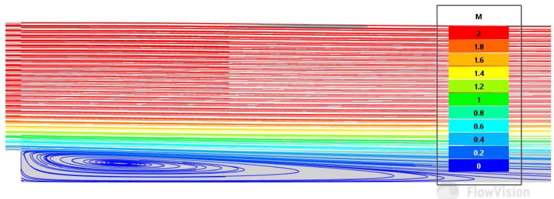
Comparison of the numerical simulation results with the linear stability theory shows that the grid resolution plays a significant role, especially for high-frequency disturbances. The observed discrepancy in disturbance growth rates at high frequencies for coarse grids results from an insufficient number of cells per wavelength. The selected time step of 20 ns gives a reliable result, and its reduction does not change the result. For a flat plate, numerical simulation in the above-described formulation gives a result close to the calculations according to the linear theory. This indicates the correctness of the problem statement and the reliability of the results.

3. Influence of Longitudinal Slots on the Development of Disturbances in the Boundary Layer

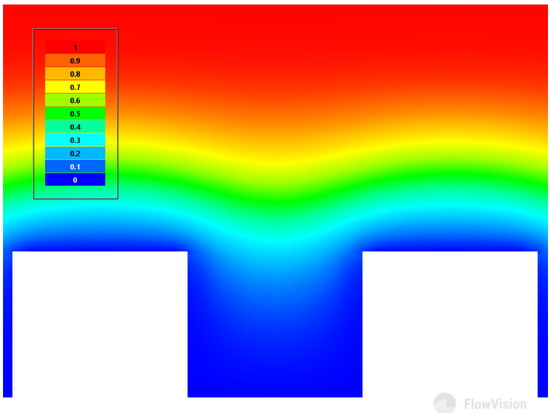
Figure 4 illustrates the features of the mean flow in the boundary layer over a plate with longitudinal slots. Figures 4a and 4b display streamlines in the xy-plane of the slot near its initiation point for slot depths of $h = 0.18$ and 0.5 mm, respectively. The color of the lines corresponds to the flow Mach number. Figure 4c presents the streamwise velocity field, normalized by the freestream velocity, in the yz-plane at $x = 80$ mm for the case of slots depth $h = 0.5$ mm.



(a) Streamlines in the slot. $h = 0.18$ mm



(b) Streamlines in the slot. $h = 0.5$ mm



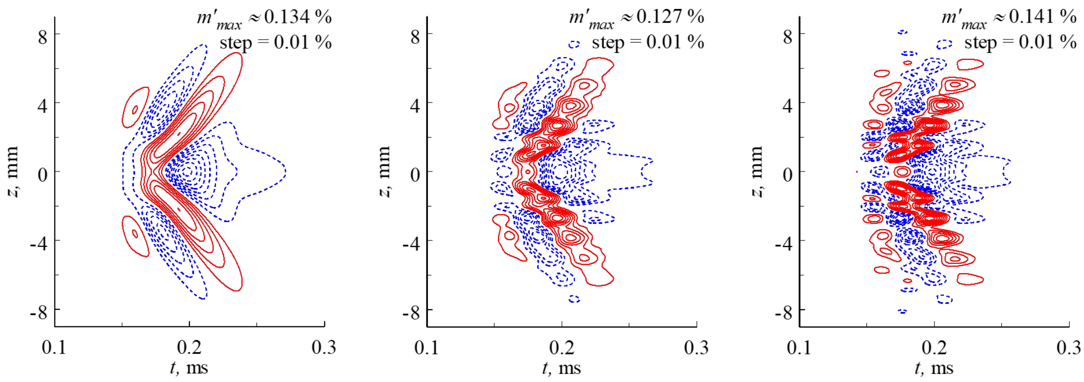
(c) Normalized streamwise velocity field.
$x = 80$ mm, $h = 0.5$ mm

Figure 4. Mean flow in the boundary layer over a plate with longitudinal slots

Near the leading edge of the slots, within the recesses, a local flow separation is observed. The streamwise extent of this separation grows with increasing slot depth but does not exceed $15 \cdot h$. Further downstream, the flow inside the slots remains attached. The flow velocity within the slots is subsonic. Transverse modulation of the boundary layer flow is evident, while the outer flow remains uniform. It can also be seen that the slot depth and width are comparable to the boundary layer thickness at this location.

Figure 5 shows the isolines of the mass flow pulsations in the zt-plane at $x = 80$ mm for the cases of a smooth plate and a plate with slots of depth 0.18 and 0.5 mm. The data are shown for the values of the y-coordinate at which the pulsation amplitude is maximum. Note that in the boundary layer above the slots, the position of the maximum disturbance level shifts closer to the plate compared to the case of the smooth surface.

In the region under consideration, the disturbance from the source has an amplitude of about 0.1%. In the presence of slots on the plate surface, the disturbance is modulated in the transverse direction. The spatial extent and structure of the disturbances generally do not change



(a) Smooth plate (b) $h = 0.18$ mm (c) $h = 0.5$ mm

Figure 5. Isolines of mass flow pulsations in the zt-plane in the boundary layer region with maximum disturbance amplitude at $x = 80$ mm

significantly. The propagation speed of the disturbances is the same in case of the smooth surface and slots.

Figure 6 shows the growth of the disturbances downstream. The dependencies of the value of S_m calculated with formula (3) are shown for the cases of the smooth surface and the plate with slots of different depths. In Fig. 6, the coordinate of the slot beginnings on the plate is marked by a vertical black dashed line.

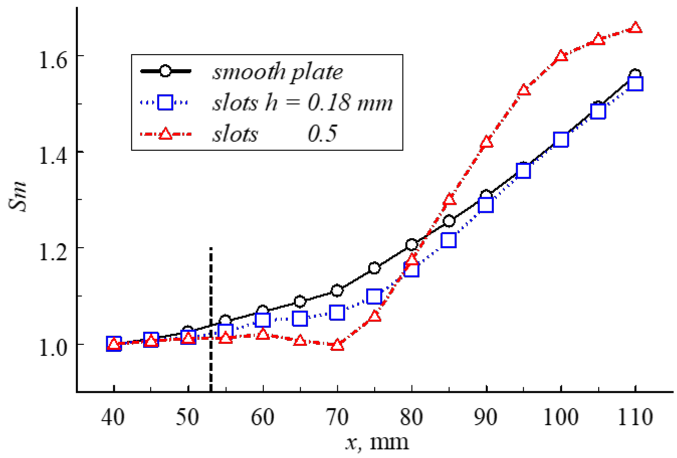
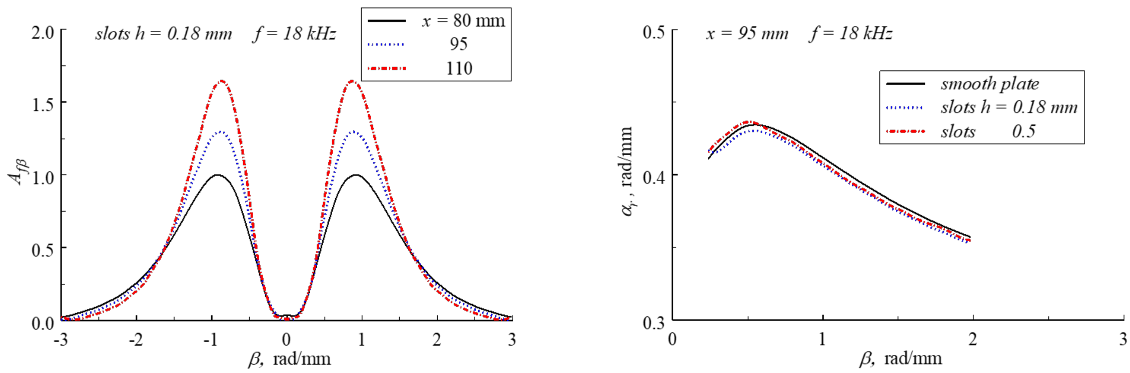


Figure 6. Growing disturbance downstream

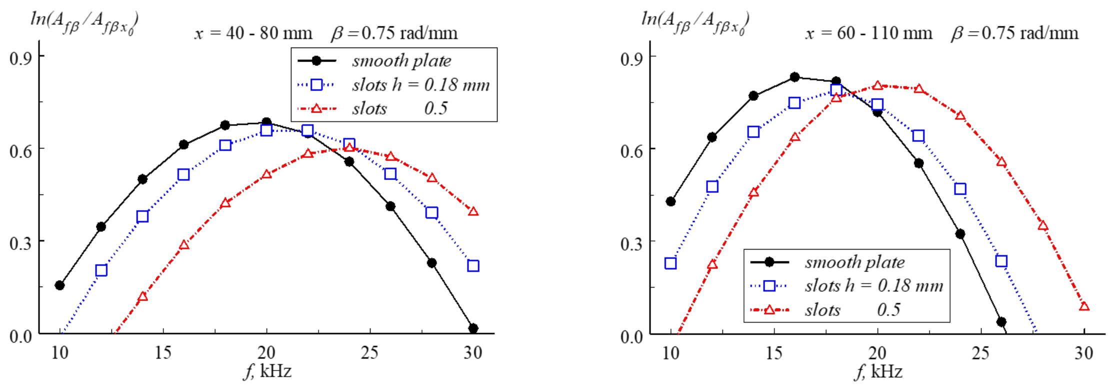
On the smooth plate, the disturbance level increases monotonically downstream. Slots lead to a stabilization of disturbance growth, especially in the area of the start of the slot. In case of 0.18 mm deep slots in the far zone ($x > 90$ mm), the disturbance growth is similar to that of the smooth plate. With increasing slot depth, the disturbance growth increases significantly in the far zone from the start of the slot.

More detailed information on the influence of longitudinal slots on the plate on the development of disturbances at the boundary is provided by frequency-wavenumber analysis. Figure 7a



(a) Wave spectra for $f = 18$ kHz (b) Dispersion relation

Figure 7. Amplitude-wave properties of the disturbance evolution



(a) $x = 40$–80 mm (b) $x = 60$–110 mm

Figure 8. Wave growth with $\beta = 0.75$ rad/mm in the region of the beginning of the slots and in the flow above the slots

shows the wave spectra of a localized disturbance for the frequency $f = 18$ kHz for the case of a flow over slots with a depth of 0.18 mm at different values of the coordinate x. Figure 7b shows the dispersion relation for the same frequency obtained for the cases of the smooth surface and the plate with slots.

The wave spectra obtained have a shape typical of a linear wave packet. The fastest growing waves have wavenumbers in the range of 0.6–1.2 rad/mm. This corresponds to wavelengths of 5–11 mm, which is a multiple of the slot period in the configurations considered. Similar spectra were obtained for deeper slots and for the case of a smooth surface. Figure 7b shows that the slots on the surface have only a minor influence on the dispersion relation. This result can be used in calculations according to the linear stability theory for cases of a plate with slots on the surface.

Figure 8 shows the effects of slots on the stability of the boundary layer. Distributions of wave growth with a wavenumber of $\beta = 0.75$ rad/mm are shown for different frequencies. The relative growth of the disturbances is analyzed in two areas: in the area of the beginning of the slots $x = 40$–80 mm, in the boundary layer above the slots $x = 60$–110 mm.

For both the smooth plate and the slotted plate, the value of the dimensional frequency of the most unstable boundary layer waves decreases downstream. This agrees with the results of the linear theory. From the numerical simulation data, the presence of slots on the plate surface leads to a change in the stability properties of the boundary layer. Initially, a clear shift in the frequency range of the unstable waves is observed. With the increasing slot depth, the frequency of the most strongly growing disturbances increases. For the slot configurations considered, the frequency of the fastest growing waves increases by more than 30% compared to the case of the smooth plate.

In the area of the beginning of the depressions (Fig. 8a), a lower growth of disturbances in the boundary layer can be observed on the plate with slots than in the case of the smooth plate. As the depth of the slots increases, the growth coefficient of the disturbances decreases. Downstream, in the flow above the slots (Fig. 8b), the difference in the growth of the disturbances in the boundary layer on the smooth surface and on the plate with slots decreases. However, there remains a clear shift in the frequency range of the unstable waves.

The data presented in Fig. 8 allow us to formulate the following explanation for the results on the growth of disturbances in physical space (Fig. 6). In the area of the beginning of the slots on the model, the growth of the disturbances is significantly lower compared to the case of the smooth surface (Fig. 8a). Therefore, in Fig. 6, a decrease of the disturbances can be observed in the range of $x = 55$–80 mm. Downstream, the growth of disturbances of the boundary layer on the plate with slots approaches the case of a smooth surface. However, in Fig. 6, for the case of a slot depth of 0.5 mm, a significant increase in disturbances can be observed compared to the smooth model. This can be explained by the fact that the slots lead to a shift of the frequency range of the unstable disturbances into the range of higher frequencies compared to the case of the smooth plate. A more intense growth of the high frequency disturbances can lead to a larger integral increase of the disturbances in the physical space. On this basis, it is possible to draw a conclusion about the influence of the investigated slots on the plate surface on the laminar-turbulent transition on the plate. A significant delay of the laminar-turbulent transition is possible if the spectrum of flow disturbances is low-frequency. However, if high-frequency pulsations predominate in the oncoming flow, early turbulization of the boundary layer is possible.

Conclusion

The influence of longitudinal rectangular slots on the plate surface on the stability of the boundary layer at Mach number 2 is investigated by a numerical simulation of the evolution of small amplitude local disturbances. The calculation formula for the evolution of disturbances in the boundary layer on a smooth plate is verified. The numerical simulation data on the growth of instability waves agree with the results obtained by the linear stability theory. It is shown that the grid resolution in the calculations mainly influences the growth of high-frequency disturbances.

The analysis of the evolution of a localized disturbance in the boundary layer of the smooth plate showed a monotonic growth downstream. In the area where the slots begin, the disturbance growth is lower than on the smooth surface. In the flow above the slots, it was found that the growth of the disturbance depends on the depth of the slots. At a slot depth of $h = 0.18$ mm, the disturbance growth is close to the value of the smooth surface, while it is significantly higher at $h = 0.5$ mm.

A frequency-wavenumber analysis of the disturbance development in the boundary layer of the smooth plate and the plate with slots is carried out. In the flow above the slots, the wave spectrum of the disturbances has the character of a linear wave packet. The dispersion relation is practically the same both for the case of the smooth surface and in the presence of slots on the plate. It is found that the most unstable waves shift to higher frequencies in the presence of slots.

The results of the numerical simulation confirm the possibility discovered earlier in experiments [11] of influencing the stability of the supersonic boundary layer of a plate by means of shallow longitudinal slots.

Acknowledgements

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